

Transition to College Mathematics and Statistics

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with

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Unit 8: Mathematics of Democratic Decision-Making

Transition to College Mathematics and Statistics (TCMS) consists of eight coherent and focused units with deliberate connections among topics across units. Each unit is comprised of two to four problem-based, inquiry-oriented, and technology-rich multi-day lessons. Each lesson consists of two to four related investigations emphasizing mathematical modeling and important mathematical practices and habits of mind.

Units culminate with a “Looking Back” lesson intended for students to review and synthesize their understanding of key ideas developed in the unit. As such, the following “Looking Back” lessons for each TCMS unit provide potential users an overview of our approach to important mathematical ideas and the expectations and nature of collaborative student work. Preceding each “Looking Back” lesson is the table of contents for the unit.

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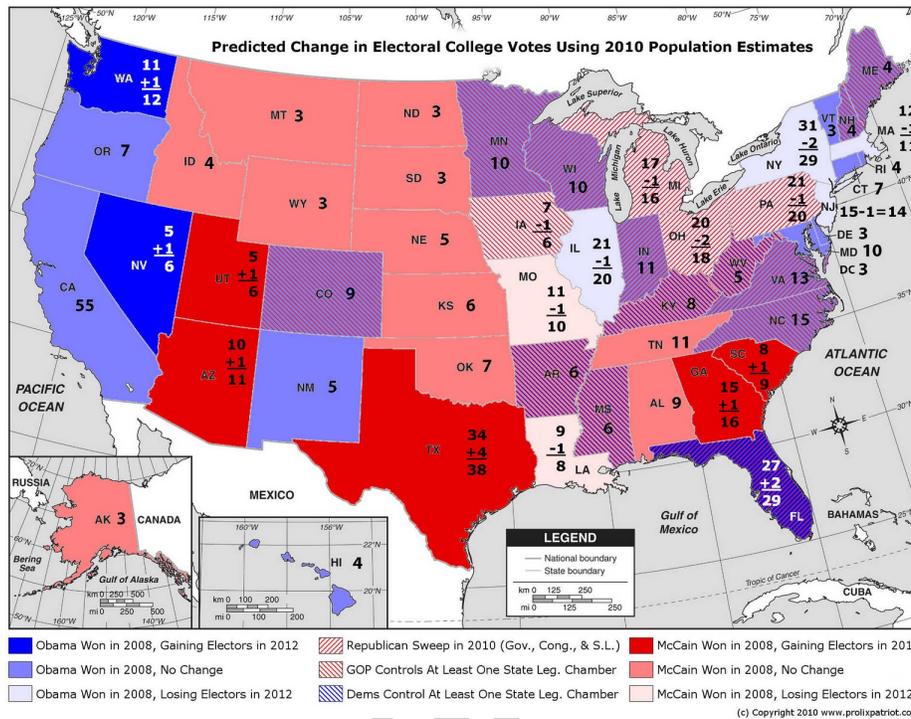
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LESSON 3 Looking Back



Source: prolixpatriot.blogspot.com/2010/12/one-state-two-state-red-state-blue.html

In this unit, you studied some mathematics of strategic decision-making in situations involving social choice, particularly in cases of voting and fair division. To help make the best decisions in these situations, you applied a mathematical analysis of ranked-choice voting, weighted voting, and fair division of both divisible objects (like a cake) and indivisible objects (like family heirlooms or seats in the U.S. House of Representatives). The tasks in this final lesson give you a chance to review and pull together the key ideas and methods of the unit.

- 1 Waterloo, Iowa, adopted an ordinance in 2002 requiring candidates for city office to get more than 50% of the vote. This first affected the election for Mayor in 2003. On November 4, 2003, the mayoral election results were as follows.

Candidate	Votes
Miller:	4,454
Hurley:	3,779
Rooff:	3,375
Abebe:	1,485

Source: "Rooff ousted from office," *WCF Courier*, 11/05/03, Tim Jamison
wfcourier.com/news/top_news/article_daf4490c-1599-5d4b-acdb-01b235e34e2f.html

- a. Who is the plurality winner? Is there a majority winner?

- b. In the runoff election four weeks after the first election, Hurley defeated Miller and was elected Mayor of Waterloo. Consider a hypothetical ranked-choice voting situation with the top three candidates, Miller, Hurley, and Roof. Newspaper accounts indicate that Roof and Hurley had similar policy positions that were markedly different from Miller’s positions. Assume Miller supporters are evenly divided in their opinions of Hurley and Roof. A possible preference table is given below.

Ranked Top Three Mayoral Candidate Preferences

	Rankings			
Miller	1	1	3	3
Hurley	2	3	2	1
Rooff	3	2	1	2
Number of Voters	2,227 voters	2,227 voters	3,375 voters	3,779 voters

Explain why this preference table could be used as a possible representation of the election description above.

- c. Do you think that the new ordinance in Waterloo, which required a runoff for this election instead of a plurality winner, resulted in a fairer outcome? Justify your answer by analyzing the preference table.
- 2 As you learned in this unit, there are several different methods for fairly dividing goods between people. Some methods are designed to work for just two people, like the Adjusted Winner method, while other methods work for any number of people, like Knaster’s procedure. You may be able to apply more than one method in a given situation.
- a. The table below shows the confidential point bids from two heirs for five items in an estate. Use the Adjusted Winner method to find a fair allocation of the items to the two heirs.

Bids (based on 100 points)

	Alanna	Bilal
Car	20	15
Pickup Truck	10	20
Jewelry	5	5
House	45	35
Summer Cabin	20	25

- b. Suppose that instead of making a points bid, Alanna and Bilal submit confidential bids for the cash value of each item, as shown in the table below. Use Knaster's procedure to fairly divide the items.

Bids (cash value in dollars)

	Alanna	Bilal
Bid for Car	20,000	15,000
Bid for Truck	10,000	20,000
Bid for Jewelry	15,000	15,000
Bid for House	450,000	350,000
Bid for Cabin	40,000	50,000

Summarize the Mathematics

There are many situations in politics, business, and everyday life in which you need to make decisions that are optimal in specific ways. In this unit, you have learned some mathematical concepts and methods that help you make good decisions. You have mathematically analyzed voting and fair division. In particular, you have studied the following topics.

1. Ranked-choice voting and related vote-analysis methods
2. Arrow's theorem
3. Weighted voting
4. Political apportionment (methods of apportioning seats in Congress)
5. Fair division of divisible and indivisible objects

Choose at least one of the topics listed above. Working with a partner, prepare a written and oral report on the topic using the guidelines below.

- Give a brief non-technical overview of the topic.
- Outline the sub-topics or details that you learned for your chosen main topic.
- Develop a thoroughly worked-out example illustrating one detail or sub-topic for your chosen main topic.

Be prepared to present your report to the class and respond to questions.

Check Your Understanding

Write, in outline form, a summary of the important mathematical concepts and methods developed in this unit. Organize your summary so that it can be used as a quick reference in your future work.